Technical Paper

How The End of Libor Will Impact Delta-1 Rates

January 2019

Jonathan Rosen, PhD

FINCAD®

How the End of Libor Will Impact Delta-1 Rates

The future of interest rate curves, pricing and risk models applied to Libor fallbacks for legacy trades based on the new family of interest rate benchmarks

Introduction

The global financial industry is beginning to make preparations for the end of the London Inter-bank Offer Rate (Libor) by Jan 1, 2021. This is certain to be a major change in the industry and will require global markets to adapt to a new crop of benchmark interest rates. The history of Libor's rate-setting mechanism is connected to the original syndicated loan facilities of the inter-bank market, and over the years Libor has become formalized and governed to safeguard its role as a credible interest rate benchmark. However in the wake of the 2008 crisis, liquidity for Libor in the inter-bank market has dried up. It can rightly be argued that for the 200 trillion in derivatives, bonds, loans, and securitizations linked to Libor, there is risk in referencing a funding rate that is not transaction-based and open to manipulation.

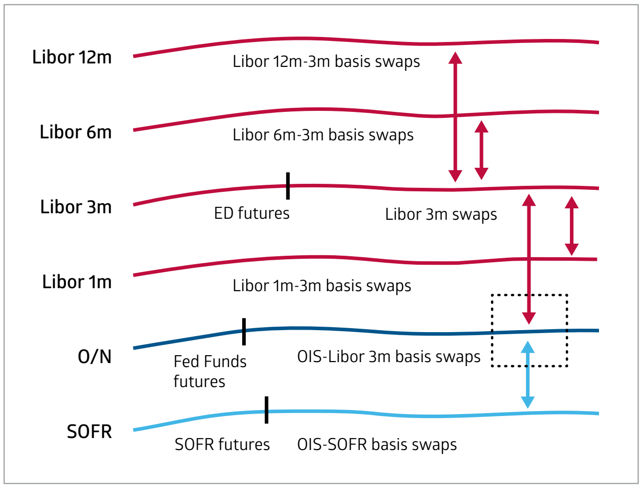
In response to these systemic problems with Libor, the statement by the UK authority that Libor rate polling will no longer be compulsory after 2021 signals the looming end of Libor as a functional benchmark rate. To prepare for the post-Libor world, the first step involves the selection of alternative benchmarks, and every country or economic zone will have their own choice – in some cases using even relatively new benchmarks, such as SOFR (USD) and ESTER (EUR), and in other cases using established overnight rates like SONIA (GBP). The first hurdle is an imminent need to grow the associated markets linked to the new rates, and as a result the industry standards for modeling interest rates curves will need to quickly adapt to these major changes in benchmark rates and their reference markets.

1.0 Interest Rate Curves in the Post-Libor Landscape

Today's advanced curves are built from an interwoven web of market data for payoffs from deposits, futures and swaps to tenor basis swaps, compounding swaps, and FX basis swaps that collectively span the rate tenor, collateralization, and currency basis – and which all generally reference Libor as a benchmark rate. If Libor disappears, there remains a need for interest rate curves for alternative benchmark rates, which can be used to price legacy derivatives as well as new trades in the funding markets. However, as we will see in the next section, the future benchmark curves will face disruption along with the disappearance of Libor, and in the following section we uncover a future with strong dependence on volume in new basis instruments taking the place of those that reference Libor today.

1.1 The Basis Tug o' War – USD Interest Rate Curve Example

In the case of USD rates, even OIS is interwoven with the fate of Libor, since the Libor-OIS basis markets will be disrupted by the end of Libor, and these are used to build the OIS curve at long maturities. Signs may point to a switch from OIS-Libor basis swaps, to OIS-SOFR basis swaps to build the USD-OIS curve for discounting trades whose collateral earns OIS rates. However this is as yet uncertain since long dated OIS-SOFR basis swaps have yet to materialize.



When it comes to how to choose a collateralized discount rate and whether OIS or SOFR is appropriate, it is theoretically about how the return on margin is calculated, which may follow the popularity of funding against treasury collateral versus unsecured overnight rates. However in terms of curve building, if OIS is a more liquid benchmark than SOFR in any range of maturities, then the tug o' war of curve building could actually use OIS and the OIS-SOFR basis to build the SOFR curve.

The wider picture is that wherever different short-term funding benchmarks share liquidity, their curves can be influenced by one another, especially at longer maturities where derivatives markets on the interest rate basis are an important source of information. In USD, when Libor slips away and the tug o' war becomes imbalanced, the whole web of curve building will need to be reshaped around the markets on SOFR and OIS.

Another basis problem may crop up related to the foreign currency funding strategy, where investors choose to fund in a foreign currency and swap back to domestic currency obligations at favorable rates. When building foreign discount curves from cross-currency swaps, Libor is used as a benchmark rate within the market data instruments. If Libor disappears, market cross-currency basis swaps could shift to reference alternative benchmark rates. There will be new risks posed on these existing funding strategies, since the cross-currency basis will span the basis between the different types of overnight benchmarks selected in different regions, for example spanning the unsecured-secured rate basis as different markets choose either secured or unsecured benchmark rates. These new basis risks will now be included in the cross-currency basis markets.

1.2 Fundamentals Connect Curve Building to the Real Markets

This section introduces the necessary concepts for connecting curve building and pricing theory, in order to consider the sum impact of Libor's disappearance on interest rate curves. The fundamental pricing theorem instructs to first identify a convenient numeraire asset, which is a traded and stochastic market variable, but if it is selected carefully we can actually ignore its future dynamics. The simplest future payment obligation is a single payment of Libor at a future time T, which can be most easily priced in the T -forward measure, relative to the forward discount bond numeraire asset:

$$V(t_0) = D(t_0, T) E_{t_0}^{Q_T}[L(T_S; T_S, T)] \tag{1}$$

From left to right, $V(t_0$ is the present value and $D(t_0, \cdot)$ is the discount curve as of the valuation date $(t_0$, $L(T_S; T_S, T)$ is the Libor rate fixed at T_S for the rate period from T_S to T, and paid at T (ignoring fixing/payment delay), with the addition of notional amount and accrual fraction which are omitted here for simplicity. If we define the forward Libor rate as follows:

$$L_F^\tau(t_0, T) = E_{t_0}^{Q_T}[L(T_S; T_S, T)] \tag{2}$$

Here τ denotes the relevant Libor tenor $(\tau = T - T_S)$, resulting in:

$$V(t_0) = D(t_0, T)\ \ L_F^\tau(t_0, T) \tag{3}$$

We can see that the present value for this future payment obligation is obtained simply by evaluating two curves at their appropriate times, the discount curve and Libor forward curve. It is by applying these same arguments to Libor-linked derivatives with known prices that we are able to infer these curves in the first place. If there is a market quote for a Libor payment fixed at T, it can be inferred:

$$L_F^\tau(t_0, T) = \frac{V_{Market}(t_0)}{D(t_0, T)} \tag{4}$$

From these arguments it can be appreciated that today we have a very convenient companionship between interest rate derivative markets and pricing models, where market prices for instruments with future Libor-linked payment obligations are used to build interest rate curves consistent with the market prices, which in-turn can also be used to price other Libor-linked cashflow obligations not found in the wider markets. This convenience rests on the fact that the market interest rate derivatives pay the Libor-linked payout, in order to directly infer the future expectation of Libor from the markets.

Notice that in order to build the Libor curve, first the discount curve is needed. Most derivatives have moved to collateralization as a way to eliminate counterparty default risk, where the rate earned on collateral is the appropriate rate to use for discounting in risk-neutral valuation, so that overnight lending rates can be used as a proxy for the risk-free discounting rate, and we build this discount curve from instruments which pay the compounded overnight rate. The simplest example of such an instrument is a single payment obligation of the compounded overnight rate, which is valued as:

$$V(t_0) = D(t_0, T)\ E_{t_0}^{Q_T}\left[\left(\prod_{k=T_S}^{T} 1 + \delta\, r_k\right) - 1\right] \tag{5}$$

$$= D(t_0, T)\ E_{t_0}^{Q_T}\left[\frac{D(T, T_S)}{D(T, T_S + 1)} \cdots \frac{D(T, T-1)}{D(T, T)} - 1\right] \tag{6}$$

$$= D(t_0, T_S) - D(t_0, T) \tag{7}$$

Here δ is the daily accrual of the overnight rate r_k at the k[th] time-point in the compounding period. In this way, both previously with Libor and here again for OIS, the interest rate curves are built from market prices, and are then used to model and to price all Libor-linked payments.

While reference has been made to OIS for the market instruments connected with the discounting curve, it is true that SOFR and ESTER are some of the candidates to take over the role of OIS as a discounting rate. In terms of SOFR-linked curve instruments, currently there is some liquidity in 1-month averaged-SOFR and 3-month compounded-SOFR futures, but long-dated swaps are not readily traded at present. However in many ways the SOFR-linked futures contracts have payouts which are similar to Fed-Fund futures and compounded OIS. Despite its relatively recent use as a reference rate, SOFR is very similar to OIS in terms of curve building.

1.3 Term Rates and the Tenor Basis

The modern world is multi-curve, with separate curves constructed for each Libor tenor of interest. This plethora of curves is necessitated by the idiosyncratic risk of lending on each tenor. However, before the 2008 crisis, the tenor basis was largely ignored and only a single Libor curve was used. In the old single-curve world, Libor rates at different tenors were simply compounded from the single-curve over the period of the loan payment. Unfortunately this was quite problematic in times of market turmoil, as the bank credit risk was significant enough to noticeably gap Libor rates to the compounded rate from the single curve. Simply put, the market rates no longer fit into one curve and to accommodate these gaps requires building multiple Libor curves using the tenor basis market, which involves the basis spread transactions needed to build all of the curves.

Besides the error of naïve compounding across tenors, the interest rate curve itself does satisfy an important need, which is to interpolate rates between those directly inferred from future fixings for market derivatives. For example, a given swap will fix Libor at periodic intervals corresponding with its payment frequency on the floating leg, but by building the full interest rate curve, it is possible to interpolate rates between these fixings which are still consistent with the market to which the curve is calibrated. Interpolation is the primary role of Libor interest rate curves today. However, removing Libor will disrupt the markets which currently provide the largest source of information in modern curve building. Since there will no longer be Libor curves calibrated across multiple tenors, the planned shift to overnight rate benchmarks leaves one big question: what happens to the multi-curve approach?

To answer this question, let us consider the new SOFR futures. As mentioned in the end of the last section, the 1m SOFR futures are very similar to Fed Funds futures with an arithmetically averaged rate payout. Just as with Fed Fund futures, we can apply a convexity adjustment to build the forward curve for the geometrically compounded rate. For discounting, the standard approach is to use both 1m and 3m SOFR contract types to build a single discount curve:

$$1 + \delta_{1m}\, SOFR_F^{1m}(t_0, T) = E_{t_0}^{Q_{T+1m}}\left[\prod_{k=T}^{T+1m}(1 + \delta\, r_k)\right] = E_{t_0}^{Q_{T+1m}}\left[\frac{D(T,T)}{D(T,T+1m)}\right] = \frac{D(t_0,T)}{D(t_0,T+1m)} \tag{8}$$

$$1 + \delta_{3m}\, SOFR_F^{3m}(t_0, T) = E_{t_0}^{Q_{T+3m}}\left[\prod_{k=T}^{T+3m}(1 + \delta\, r_k)\right] = E_{t_0}^{Q_{T+3m}}\left[\frac{D(T,T)}{D(T,T+3m)}\right] = \frac{D(t_0,T)}{D(t_0,T+3m)} \tag{9}$$

This is a useful approach to build the single-curve $D(t_0,\cdot)$ for discounting at the risk-free rate across many different asset classes in risk-neutral valuation. This stems from the curve instruments in theory forming a static replication strategy for a zero coupon trade. However when Libor disappears, there is a major shift in the role of the SOFR curve, in that it is no longer just for discounting, but also a reference-rate for payment obligations.

Going back to the monthly and quarterly futures, a close look at expressions (8) and (9) for the expected rates that make up the two curves reveals that there is no way to exactly recover the quarterly rate from the monthly curve and vice versa, without a model-dependent and volatility-dependent convexity adjustment. So while it is sensible to combine monthly and quarterly futures into a single curve used for discounting, when it comes to forecasting the forward payment rates, we are faced with the reality that these must be two different curves. Despite the absence of risk to bank credit, we do not actually have a single-curve for forward rates in the case of overnight rates like SOFR, OIS, SONIA, and the other alternative benchmarks. The tenor basis is still important beyond pure discounting, and practitioners will need to ensure that their models account for the tenor basis for forward benchmark rates, even possibly in the absence of basis markets and term rates when Libor is discontinued.

2.0
Libor Fallbacks and Legacy Delta-1 Instruments

2.1 What are Libor Fallbacks?

In the wake of the announced end of Libor, the industry is mobilizing to replace Libor with alternative interest rate benchmarks. The ultimate goal is to completely remove reference to Libor from trades in the global markets - a major undertaking. Now there is an imminent need to decide upon a method for removing Libor from legacy trades when Libor is discontinued. This is the realm of the "fallback", which is a contractual mechanism for amending Libor-linked trades with alternative payment calculations linked to the new benchmark rates. This is a very active and uncertain area at the time of writing, with industry consultations in circulation for derivatives (ISDA), bonds and loans (ARRC), and forthcoming for other asset classes.

So far the fallback proposals have laid out a number of options for alternative rate adjustments, spread methodologies, and contractual terms varying from open-ended amendment clauses to hard-wired rate fallbacks, including reference to term rates which don't yet exist. Amid the plethora of choices and opinions on what Libor fallbacks could look like, the question remains: what is the difference between the proposed Libor fallbacks?

To begin to answer this question, we start with an analysis of several proposed "adjusted risk free rates" ($aRFR$) discussed in the December 20, 2018 ISDA consultation results (**link**). Four specific proposals are made on how to calculate the payment rate fallback that will replace Libor in legacy trades, which are briefly:

1. **Spot Overnight Rate:** This appears to be the simplest proposal, which directly replaces Libor with the overnight rate. Its potential advantages are claimed to be its ease to understand and implement, but disadvantages boil down to it being drastically different than Libor, as we will see below.
2. **Convexity-adjusted Overnight Rate:** This approach instead replaces Libor with a functional form of the overnight rate which is intended to look more similar to a term-rate by approximating the effect of compounding the overnight rate. However it is only an approximation to a compounded rate and there are potential situations where this is not a very good approximation to make.
3. **Compounded Setting in Arrears Rate:** This rate is already in use for instrument payout calculations that are common today, such as overnight index swaps (OIS). The main disadvantage here is the way it is fixed is much different than Libor. The compounded overnight rate calculation requires overnight rate fixings throughout the payment period, when Libor would have already been fixed, thus delaying absolute certainty of the rate until immediately before it is to be paid.

4. **Compounded Setting in Advance Rate:** This is very similar to (3), except the compounding period is shifted back to the tenor-window immediately before the standard Libor fixing date to have the same fixing behavior as Libor at the start of the period. It is then adjusted as in proposal (2) to offset convexity adjustments.

Evident in these proposals is the total lack of a term rate to replace Libor, which is another big issue that is separately under consultation. These proposals are all attempts to work directly with established overnight rates and to formulate a satisfactory replacement for Libor which can be written into fallback clauses in derivatives and other financial contracts. So which proposal really is best? We have to consider a couple of different points of view in this regard, specifically here for Delta-1 products:

1. **Calculation of payment rates for immediate coupons:** All of the four proposals add clarity in how to calculate their payment obligations for legacy trades in the event that Libor goes away, and in this respect they are all clearly defined calculations which are transparent and make good strides towards avoiding uncertainty or disputes over the payment rates. There are some differences about when the payment calculation can be done, in particular the compounded in arrears differs the most significantly from the Libor fixed-in-advance which is standard today, by delaying the calculation until just before the payment. The effects on the immediate payment rate calculations are summarized in the following table:

Adjusted RFR Method	RFR Fixing Times
Spot Overnight Rate	Same as Libor (in advance)
Convexity-adjusted	Same as Libor (in advance)
Compounded Setting in Arrears	Daily Over Contract Rate Period
Compounded Setting in Advance	Daily Over Preceding Rate Period

Operationally, Libor fixing at the start of the rate period is an important feature of pricing and risk management, and it is evident that the advance and arrear compounded RFR fallbacks represent a major change in the way that rate fixings are used. In many ways the Libor in advance fixing is convenient because it allows plenty of time for the fixing to be agreed and known in advance of the payment, which is even required by some standard derivatives such as forward-rate agreements. The compounded RFR would involve the need to monitor and manage many daily fixings for every coupon paid.

2. **Valuing future payment obligations:** While the fallbacks are easily interpreted as rules for calculating the payments themselves, when these payments are scheduled at a future time, there is suddenly a need to calculate the expected future payment amounts in present value terms. In the case of Libor, this is typically done with interest rate curves for the forward rates. However, for the fallback rates which are calculated from risk-free overnight rates, it is not immediately clear how to apply curves for pricing future payments. The next section will embark us on a journey to understand the impact of the choice of fallback used on the pricing of legacy delta-1 payouts when Libor is gone.

2.2 The Present Value of Libor Fallbacks

Now we may consider the question of how to price trades with fallbacks. If there are no market prices available for cashflow obligations linked to these fallback clauses, we won't have such a convenient companionship of market prices and curves as we did for Libor. So consider the forward fallback rate:

$$E_{t_0}^{Q_T}[\,aRFR(T_S; T_S, T)] = ? \tag{10}$$

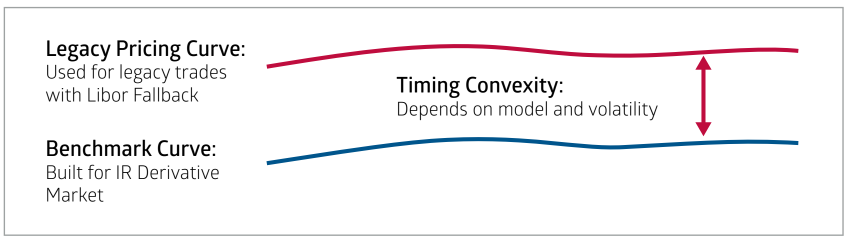
As stated above we don't seem to have a curve to help us answer this question directly, but recall that the arrears compounded fallback rate is basically the same as the OIS payout. If we take this as a baseline, we can isolate the tricky part of this calculation:

$$E_{t_0}^{Q_T}[\, aRFR(T_S;T_S,T)] = E_{t_0}^{Q_T}\left[\prod_{k=T_S}^{T}(1+\delta\, r_k)\; - \left(\prod_{k=T_S}^{T}(1+\delta\, r_k) - aRFR(T_S;T_S,T)\right)\right] \quad (11)$$

$$= \left(\frac{D(t_0,T_S)}{D(t_0,T)} - 1\right) - \,\gamma_\tau(T) \quad (12)$$

$$\gamma_\tau(T) \equiv \; E_{t_0}^{Q_T}\left[\left(\prod_{k=T_S}^{T}(1+\delta\, r_k) - aRFR(T_S;T_S,T)\right)\right] \quad (13)$$

Valuing a payment linked to the fallback rate can be divided into two components - the first just uses the discount curve, while the second is an unknown quantity $\gamma_\tau(T)$, which is generally non-zero except for the arrears compounded fallback and depends on the rate tenor τ. The quantity $\gamma_\tau(T)$ is a convexity-adjustment, which is the effect of the non-linear expectation of the difference between the payment we know how to price off of curves, and the expected payment linked to the fallback rate. Since there is no curve for $\gamma_\tau(T)$, it is necessary to model this quantity, and the calculation of $\gamma_\tau(T)$ is volatility-dependent as well as model-dependent.



2.3 Potential for a Convexity-Adjustment Arms Race

Today's reconciliation against counterparty valuations can already face challenges around different modeling choices for curve building, but soon the legacy trade fallback rates could add a completely new paradigm in volatility-modeling dependence for delta-1 rates with the convexity adjustment $\gamma_\tau(T)$, only furthering the potential for major disputes around valuation. Even with satisfactory benchmark curves after Libor, there could be a need to handle additional curve complexity, since some of the contract fallback options will make it much harder to price instruments by requiring convexity adjusted benchmark curves. For fallback options which do not involve rates paid at their natural time, timing convexity may require term structure models even for delta-1 instruments. Another way to look at this is if prices are quoted for trades with fallback payoffs, the curves built from these quotes are not the same as the curves built from standard payoffs based on risk-free benchmarks like OIS. The basis between these curves will be model dependent, and require volatility modeling for interest rates and come with a significant potential for disputes and discrepancies.

Due to these problems, it is unlikely that fallbacks will be chosen which have convexity. If this does become the new normal, then there would be an arms race in convexity adjustment and volatility modeling in order to come up with fair pricing. Practitioners would need to adapt to this convex world or be left behind, experiencing major pricing discrepancies against more sophisticated counterparties which are entirely analogous to the Libor in arrears issues that were used to take advantage of uninformed investors in recent years (**link**).

2.4 Fallback Spread Methodologies

To complete the analysis of pricing future payments linked to fallback rates, we need to consider the spread adjustment proposals in the ISDA consultation. After replacing Libor with the adjusted benchmark rate, the proposed fallback language specifies that a spread is then added to correct for the difference between the Libor fixing and the $aRFR$. All of the proposed spread adjustments are static in the sense of excluding any effects of spread volatility, and generally have a fixed definition over time once they are initially determined at the Libor discontinuation date.

Beginning with the definition of the forward adjusted risk-free rate, $aRFR_F(t_0, T)$, which is an approximate forward rate that ignores any volatility-driven convexity adjustment, the proposals put forth three potential spread adjustments which are to be added to the fallback rate. These can be described using the following definitions for the spread curve $\Delta(T)$:

1. **Spot Spread:** This method simply uses the (constant) spot spread Δ_0^τ, that is the difference of the adjusted benchmark and Libor fixings at the time Libor is discontinued.

$$\Delta(T) = \Delta_0^\tau = L_F^\tau(t_0, t_0) - aRFR_F(t_0, t_0) \tag{14}$$

   This has the advantage of simplicity, though it is very different from the term-structure of spreads that is seen today when comparing the Libor and $aRFR$ curves across maturities, and is sensitive to the market conditions at the Libor discontinuation date.

2. **Forward Spread:** This method takes the full forward curves for Libor and the adjusted benchmark and computes their difference as a spread curve across all maturities.

$$\Delta(T) = \Delta_F^\tau(T) = L_F^\tau(t_0, T) - aRFR_F(t_0, T) \tag{15}$$

   The advantage of the forward spread is that despite it being static once it is determined, it is still the spread adjustment which is most similar to the existing spread term structure currently observed between alternative benchmarks and Libor. The forward spread is also highly dependent on the market conditions at the time Libor is discontinued, which could be reason to suspect that it could be influenced by market manipulation at that time.

3. **Historical Mean/Median Spread:** This is a historical spot spread, $\Delta_{0,historical}^\tau$, which is similar to the previous spot spread, but is averaged from a history of spread data over 5-10 years, and is to be phased in over a one year transition period starting with the initial spot spread Δ_0^τ, resulting in:

$$\Delta(T) = \begin{cases} t\left(\Delta_{0,historical}^\tau - \Delta_0^\tau\right) + \Delta_0^\tau, & 0 \le t < 1 \\ \Delta_{0,historical}^\tau, & 1 < t \end{cases} \tag{16}$$

   The advantage of the historical spread method is that it does capture the market conditions at the Libor discontinuation date, but then gradually transitions to the long-term average spread behavior. The downsides are that it is still a flat spread, so that it does not capture the term-structure of spreads observed today, and its calculation is also very data intensive since it requires many years of historical data to be managed as part of its calculation.

2.5 Breaking Down the Pricing Impacts

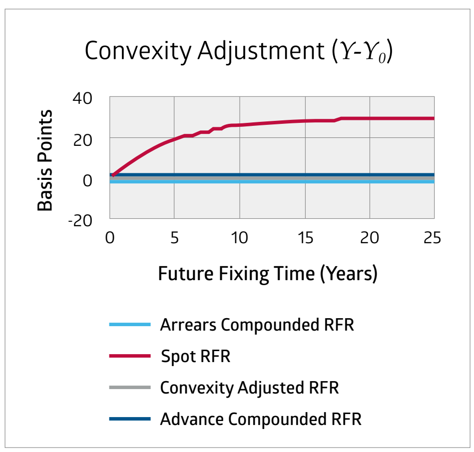
In order to understand the impact of the adjusted benchmarks and spread methods proposed, the previous analysis can be combined together, to directly compare the last Libor forward curve to the alternative benchmark and spread found in the contract fallback language:

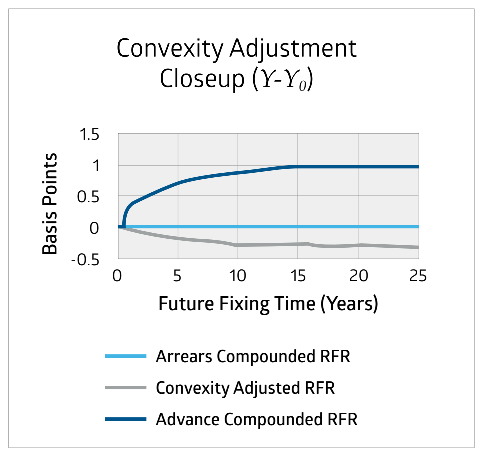
$$L_F^{\tau}(t_0,T) = E_{t_0}^{Q_T}[aRFR(T_S;T_S,T)] + \Delta_F^{\tau}(T) + \big(\gamma_{\tau}(T) - \gamma_0(T)\big) \tag{17}$$

$$\gamma_0(T) \equiv \left(\frac{D(t_0,T_S)}{D(t_0,T)} - 1\right) - aRFR_F(t_0,T) \tag{18}$$

Note that γ_0 is introduced to adjust for the difference in the forward curves between the arrears compounded risk-free rate and the fallback rate chosen. If interest rate volatility is zero, then $\gamma_{\tau} = \gamma_0$, and both are zero if the arrears compounded fallback is used in the contract fallback language.

Since Libor is the underlying of interest rate options today, current models rarely have a calibrated volatility model for overnight benchmark rates. However, just to demonstrate the size of the convexity adjustment for the various fallback rates, the results of Hull-White simulation with constant volatility of 5% and mean reversion of 10% are shown below for a 3m tenor:





The main result is that all fallbacks except arrears compounding have non-zero $\gamma_{\tau}(T)$. This pricing convexity adjustment is present to varying degrees in all but the "natural" fallback rate (arrears compounded). Clearly the direct use of the spot rate fallback is the most problematic, since it leads to a very large convexity adjustment. The size of the effect for the adjusted spot rate and advance compounded spot rate is within a basis point, but note that it appears to usually have an opposite sign, meaning that they work in different directions and could benefit one counterparty versus the other for a legacy deal. The specific numbers here are representative, but they are not the final word, as the exact numbers will depend on the level of overnight rate volatility, and the choice of mean reversion parameter – and more generally these numbers will depend on the volatility model that is used to calculate the convexity adjustment.

On the other hand, the spot and historical spread adjustments both amount to replacing the forward spread with a flat spread. This will lead to a discrepancy between the spread-adjusted $aRFR$ and the last Libor curve. This means the spread methodologies (1) and (3) will result in a jump in the present value of Libor-linked trades on the Libor discontinuation date, when fallbacks are triggered.

The spread between Libor curves and risk-free rates is a detailed measure of the market's expectation of the added risk embedded in Libor. There are many different types of risk, stemming from volatility, credit default, liquidity, and other factors related to funding against Libor. The error in ignoring the term spread between the $aRFR$ and Libor can have just as significant an impact as convexity on the present value of trades, and the error will vary significantly with maturity and also with the shape of the curves at the time Libor ends.

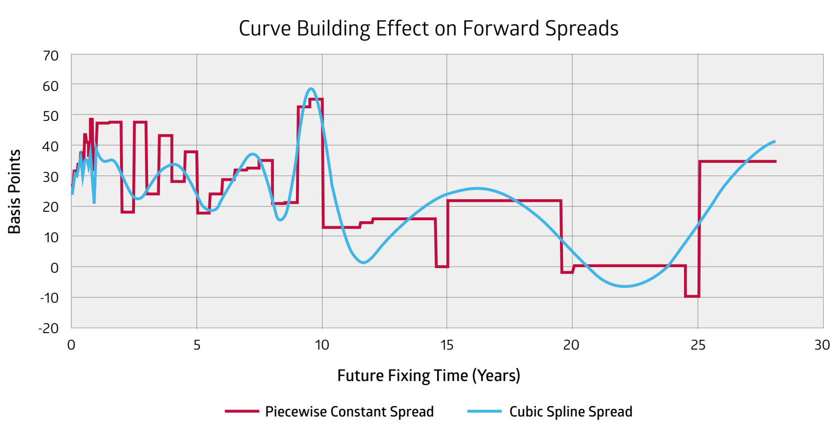
The consensus from ISDA is to favor the historical flat spread, a decision which seems to center around the relative simplicity of a historical spread and robustness to market manipulation. However, a spot spread applied to a future payment obligation is artificial from a pricing standpoint, and the present value of the future payment obligation will be significantly impacted. A significant drawback of a flat spread is that it causes an immediate change in present value, which will lead dealers to immediately call winners and losers across every position when Libor ends.

2.6 Modeling Challenges Behind the Spread Methodologies

There are two main types of spread adjustments, spot and forward, and the choice between these two will impact the value of trades at the time Libor is discontinued. The simpler, less data-intensive, and arguably less manipulation-prone choice of spread methodology is the historical mean spread. But as soon as Libor goes away, the break-even payment rate for a future payment will become the adjusted risk-free rate plus a flat spread that is independent of maturity. This is so different from Libor that the fair value of any Libor-linked payment could change the very instant the fallback language is triggered. On the contrary, the forward spread mimics the legacy Libor payoff as closely as possible at the Libor discontinuation date, avoiding any drastic change in fair value by basing the spread directly on the Libor and $aRFR$ curves. However, while the forward spread method is similar to the observed term-structure of Libor-$aRFR$ spreads, there also seems to be some modeling challenges behind it.

Today's Libor curves can be built in many different ways, with different modeling assumptions that span from simple to more sophisticated curve building methods. The primary modeling choice for curve building pertains to the choice of the interpolation method, such as using cubic splines instead of simple piecewise constant rates. If the forward spread curve is determined at Libor's discontinuation date by the difference between Libor and alternative benchmark curves, then the shape of the spread curve will depend strongly on how these curves are built.

A lurking consequence of the modeling choices behind curve building used to determine the forward spread curve, is that a standardized calculation of spread curves may not be consistent with the internal-model curve building methods used for Libor today. In order to agree prices for legacy trades once Libor is gone, standardized spread curves must be applied consistently by all parties. This will actually require an unprecedented level of standardization to take place over the internal-model curve building methods, otherwise internal models for benchmark rate curves will lead to spread curves which are inconsistent with the results of third party spread curves. An example is shown here for the spread determined from two different curve building methods which are currently standard practice:



Presently there tends to be less focus on the shape of overnight rate curves that are used primarily for discounting, and it is more common to see simpler models behind the risk-free rate curves. However, with the trigger of fallback reference rates and overall shift to transaction-based rates, the new normal for overnight-rate curve building could see a shift to more advanced methods, making it harder for counterparties to agree on a forward spread curve that is consistent with their internal models.

Other modeling challenges are present for all of the proposed spread methods. One subtle impact appears with the end of Libor also bringing a paradigm shift in the future dynamics of the present value of amended contracts, simply due to the payouts falling back to reference a combination of alternative benchmarks plus static spread curves. The sum of these two curves will combine the shape of both curves into the final pricing curve – and a summation of an interpolated RFR curve and another interpolated forward spread curve will not have exactly the same shape as a single interpolation curve used for today's Libor curves. Since the summation is built into the definition of the fallback, this effect is implicit in the summation of interpolation curves and can't be avoided without radically changing the way benchmark rate curves are constructed.

2.7 Looming ISDA Decisions, Alternative Spreads, and Risk Exposure

There is nothing sacred about the ISDA proposals for adjusted benchmarks and spread methodologies in fallback language, since they are actually just a few of the possibilities imaginable for what could replace Libor. However the role of ISDA as an industry body means that they can amend the OTC master agreement with specific fallback language that could have far-reaching effects. The analysis of the current fallback proposals highlights some of the potential problems around adjusted rate convexity and spread adjustment present value gaps, which can impact even extremely simple delta-1 trades.

All but one of the adjusted risk-free rates leads to some convexity adjustment, which can either be ignored, or else advanced volatility modeling will be needed for all legacy delta-1 trades – surely a tough pill to swallow given the widespread use of simple curve-based pricing today. Clearly the arrears-compounded fallback rate is the winner when it comes to avoiding a tidal wave of modeling complexity, despite its drawback of delaying the rate fixing relative to payment.

However in the case of the proposed spread adjustments, it is much less clear what is the best choice. A constant spread may be the preferred choice due to simplicity and robustness, but overall its major drawback is the sudden jump in the present value when fallbacks are triggered. On the other hand, the forward spread curve does not have this drawback, but instead is subject to the market situation at Libor's discontinuation. This makes the forward spread overly problematic in terms of risk to manipulation, as once this curve is set it must forever be used for the life of the trade. One question to consider is whether there is a best of both worlds: a simple, constant spread which can also eliminate present value jumps while simultaneously remaining robust to market manipulation?

One idea is to consider pricing a Libor leg under fallback terms, and determining a flat spread that is present value neutral:

$$V_{leg}(t_0) = \sum_{i=1}^{M} \alpha_i \, D(t_0, T_i) \, L_F^{\tau}(t_0, T_{i-1}) \rightarrow \sum_{i=1}^{M} \alpha_i \, D(t_0, T_i) \, (aRFR_F(t_0, T_{i-1}) + \Delta_M) \tag{19}$$

$$= \sum_{i=1}^{M} \alpha_i \, D(t_0, T_i) \, (aRFR_F(t_0, T_{i-1}) + \Delta_F^{\tau}(T_{i-1})) \tag{20}$$

$$\Delta_M = \frac{\sum_{i=1}^{M} \alpha_i \, D(t_0, T_i) \, \Delta_F^{\tau}(T_{i-1})}{\sum_{i=1}^{M} \alpha_i \, D(t_0, T_i)} \tag{21}$$

This constant spread Δ_M can be referred to as the "swap-rate spread" method, since it is determined much the same as the swap rate, using the forward spread as the floating rate. It is specific to the deal, since it depends on the payment schedule accruals α_i and the maturity M of the leg, and it reduces to the forward spread for the case of a single payment obligation. It should be calculated based on the discount curve at the time Libor ends and held constant afterwards. Additionally, the historical average can be used to increase robustness to manipulation:

$$\Delta_{M,historical} = \frac{\sum_{i=1}^{M} \alpha_i \, D(t_0, T_i) \, \Delta_{F,historical}^{\tau}(T_{i-1})}{\sum_{i=1}^{M} \alpha_i \, D(t_0, T_i)} \tag{22}$$

$$\Delta(T) = \begin{cases} t\left(\Delta_{M,historical} - \Delta_M\right) + \Delta_M \,, & 0 \le t < 1 \\ \Delta_{M,historical} \,, & 1 < t \end{cases} \tag{23}$$

Despite this spread being deal-specific, there is no sudden impact on the value when Libor is discontinued, and it is generally no more difficult to calculate than the forward spread and historical spread. However if it is important to preserve the condition of exactly matching payments between deals at different tenors, then there is a potential issue that there will be residual mismatch of the spreads across different trades. The maturity-specific flat spread adjustment is not part of the ISDA consultation or any other proposal known at the time of writing, but it is does demonstrate that it is possible to devise constant spreads which can eliminate the present value jump when Libor is discontinued. When it comes time to negotiate fallback terms with a counterparty, it may go much more smoothly if the present-value cliff effect is not part of the equation.

One more related topic remains for delta-1 products: how will the risk exposure be affected by the Libor fallbacks? Overall it is straightforward that sensitivities will shift from the Libor curve risk factors to those of the RFR curve, and even potentially the RFR volatility if there really is a non-zero convexity adjustment. If Libor risk is already perfectly hedged, then not much will change for delta-1 instruments when Libor is discontinued, as long as all hedging instruments use the same adjusted rate as a fallback. On the other hand, any outright Libor exposure would be converted into RFR exposure in a straightforward way.

However if there are any mismatches between the adjusted RFR used, for instance differences between derivatives, loans, bonds, or securitizations, then multi-asset portfolios could be in for unexpected changes in risk profile. The details of these effects are hard to gauge without a specific portfolio composition, and a specific scenario in mind regarding mismatched fallbacks. There is arguably some potential for these effects across different product types based on the outcomes of proposals in circulation and other bi-laterally renegotiated terms of legacy contracts. The main takeaway is this will require careful analysis when determining hedge efficacy and risk limits under the proposed fallbacks.

2.8 Customized Exposure with Structured Spread Index

Up to this point the spread adjustment methodologies have been laid out in order to show how to avoid a sudden jump in present value at the end of Libor. But a more interesting terminus to this discussion is that a structured spread calculation could actually give just about any risk profile after Libor. Here a bold and different picture of fallback terms to consider is one instead tailored to the specific needs of individual investors. The static spread approaches are easy to standardize for all participants, but they crudely remove risk factors to the Libor- RFR basis, thus eliminating market risk to bank credit and the term-lending basis. Individuals should not hope to have much influence over the choice of alternative benchmark rates in their legacy trades since these are transaction-based, but the spread adjustments themselves could be structured products, where exposure to any risk factor, such as sector indices or other observable indicators, can be built-in.

A structured spread could derive from any of the previously discussed spread proposals, and add on an index-based term:

$$\Delta(t) = \Delta_M + I(t) - I(0) \tag{24}$$

Here $I(t)$ is the chosen exposure index at time t, and $I(0)$ is the index level when the fallback provision is triggered. By using the "swap-rate" spread and basing the index value at the time of the end of Libor trigger, this spread method does not result in any issue with present value cliff effect. What it does achieve is building in exposure to the index into the contract payout formula. If this spread is used for an interest rate swap fallback provision, it is economically similar to a portfolio of an overnight index swap based on the risk-free benchmark rate, a fixed leg with rate Δ_M, and an index swap for the chosen exposure index. Valuation of the index payout is just like an index swap.

Let us turn to a practical example of this idea to suit the needs of an investor who does not want to eliminate market risk to bank credit when Libor is discontinued. While many market participants are happy to hedge and eliminate exposure to Libor, it is also possible that some trading strategies require Libor exposure, such as speculation on the Libor-RFR basis. If a credit default swap rate is available, which is a good proxy for the bank credit risk, then this can be used as the exposure index, resulting in a payout which is economically similar to a portfolio including a constant-maturity credit default swap and which reincorporates bank credit risk into the fallback payout.

This presents an alternate ending to the story, with the potential to design risk exposure to individuals' needs when Libor is discontinued. This is the result of the mainstream spread methodologies that freeze the Libor-RFR basis into a flat spread rate, and offers the possibility to subsequently swap this fixed spread into a floating exposure with an index swap. This type of fallback is a more complex structured trade, but is very similar to a portfolio with an index swap and otherwise doesn't present any significant valuation challenges. It also provides the customized exposure profile that is desired after the end of Libor. There could be a lot of interest from investors for specific risk profiles replacing Libor that better suit their needs, and the counterparties who will offer these structured index swaps could become very popular in the days leading up to the end of Libor.

Conclusion

It is clear that for a multitude of reasons the end of Libor is an enormous change, with the potential for widespread impacts on interest rate markets, since Libor is so heavily ingrained in existing financial contracts, derivatives markets, and pricing models. One example of this is the illustration of the interconnected basis markets and their role in curve building in the modern multi-curve models. The end of Libor deadline is set for 2021. But it is not clear when term rate basis markets will emerge for the alternative benchmarks, which means that the basis markets may not be available to provide input into curve building in the near future. This would undoubtedly lead to serious complications for multi-curve modeling.

It is also evident that none of the proposed fallbacks, new benchmarks, or spread adjustments will capture the market risk of default for unsecured debt the way that Libor does today. The real economic Libor-RFR basis will indeed change over time after the transition away from Libor polling, and to price uncollateralized payment obligations such as commercial loans, the pricing models in use today will need to grow beyond the static spreads negotiated for legacy deals, and become significantly more advanced in terms of counterparty risk pricing. The market consensus for creditworthiness of a counterparty can be estimated from other sources, such as bond yields or default swap spreads. To discount unsecured debt may require credit scaling and recovery assumptions from liquid debt or credit derivatives on the inter-bank sector which can facilitate CVA calculation for unsecured term loans. This means CVA could take the place of the Libor discount curve in the future, and the incorporation of credit derivatives into commercial loan pricing will require increased sophistication for modeling in the future.

However, there is an immediate concern around making preparations with fallback provisions in legacy contracts, since in most cases the mechanism for how to handle a loss of Libor fixings is not very clear. There is widespread consensus for the winning fallback for Libor to be the arrears compounded risk-free rate, since this matches well with existing overnight derivatives markets and avoids convexity effects. While these convexity effects may be on the order of a basis point in some cases, they are sufficiently worrying in part due to their systematic impact on every floating rate exposure.

On the other hand, the modeling challenges with forward spread methodologies have pushed the industry towards simpler and more robust historical spot spread adjustments. However, this method will lead to a valuation jump at the end of Libor. This is also a concerning effect, and the jump in value is a significant risk imposed by the industry consensus historical spot spread adjustment. It is shown here that this is a somewhat artificial risk, since it is possible to devise a spot spread with no valuation jump on a contract by contract basis, and investors should strongly consider how they will manage and even begin to price in these valuation jumps for legacy trades at the end of Libor transition.

The industry fallback decisions will have an impact on both valuation and risk exposure at the end of Libor. In terms of risk, the primary effect is for Libor exposure to transfer onto the alternative benchmarks, and there is the systematic exposure of the entire portfolio to the historic spread which may be significant. But beyond the end of Libor horizon is the reality that any outright exposure to the Libor-RFR basis or Libor tenor basis will be frozen out of legacy contracts. Practitioners should pay attention to the possibility of using structured index swaps or possibly even negotiating customized spread calculations into their contract fallbacks in order to have more control over their risk profile after the end of Libor.

Disclaimer

FINCAD makes no warranty either express or implied, including but not limited to, any implied warranties of merchantability or fitness for a particular purpose regarding these materials, and makes such materials available solely on an "as-is" basis. In no event shall FINCAD be liable to anyone for special, collateral, incidental or consequential damages in connection with or arising out of purchase or use of these materials. This information is subject to change without notice. FINCAD assumes no responsibility for any errors in this document or their consequences, and reserves the right to make improvements and changes to this document without notice.

Copyright

Trademarks

F3™ (Patented Technology), FinancialCAD® and FINCAD® are registered trademarks of FinancialCAD Corporation. Other trademarks are the property of their respective holders.

Revisions

Every effort has been made to ensure the accuracy of this document. FINCAD regrets any errors and omissions that may occur and would appreciate being informed of any errors found. FINCAD will correct any such errors and omissions in a subsequent version, as feasible. Please contact us at **info@fincad.com**.

Vancouver
New York
London
Dublin

fincad.com

FINCAD®

About FINCAD

FINCAD is the leading provider of enterprise portfolio and risk analytics for multi-asset derivatives and fixed income. An industry standard since 1990, our advanced analytics, flexible architecture and patented technology enable better investment and risk management decisions. We are committed to helping our clients solve their toughest challenges, without compromise. Clients include leading global asset managers, hedge funds, insurance companies, pension funds, banks and auditors. To learn more about FINCAD's award-winning solutions, please visit fincad.com.

Contact Sales or Support Toll-Free:

USA/Canada 1 800 304 0702 **Europe** 00 800 4400 5060 **Email** info@fincad.com